**Body postion estimation of legged robot**

The explain of the body position estimation of the legged robot based on IMU, Leg odometry and extra position sensor.

1. **State prediction model**

The system state variable is：

，

where  represent the absolute position of the body, the absolute linear velocity of the body and the absolute position of the foot. ( is the number of leg branches).

Equation of state：



where  is the cycle time，，， is the body acceleration (IMU) in the absolute coordinate system.

Noise: where  are the noise coefficient.

1. **Observation model**

Observed variable：



where  represents the position and linear velocity measured by the external position sensor， and  represent the absolute velocity and absolute height of foot.

Observation model：



1） can be obtained through the forward kinematics of the robot with one leg.

2） can be obtained the forward kinematics of the single leg of the robot, the observed quantity is expected to obtain the absolute velocity of the fuselage through the supporting leg, so there should be no observation in the swinging state.

3）

where 

Noise where  are the noise coefficient.

1. **Model optimization**

Whether a single leg supports the ground face has a great influence on the state estimation of a multi-legged robot during walking. A support confidence parameter  was designed according to the gait design and leg branch motion. Under ideal conditions, the confidence parameter of support state is 1, and the confidence parameter of swing state is 0.

Covariance and observed variables are adjusted for confidence parameters:

1. Optimization of observation variables

The body observed is in a swinging state relative to the speed at the end of the foot (the absolute speed of the body), and the fuselage speed is not accurate. The body velocity observed by the leg branch is optimized as follows：



where  body velocity observed for the leg.

When the robot is supported, the absolute height of the foot end does not change under the condition of non-skid:



where  is the height at which the end of the leg last touched the ground. In particular, it can be set  when walking on flat ground.

1. Optimization of covariance

When the end of the leg branch is in the supporting state, the reliability of all state prediction and observation is high, but the confidence of the end position predicted by the swing state is low, and the confidence of the observed end velocity and height is very low.

Assuming a constant noise , a class of common matrices is constructed:



Then the two noise optimizations can be expressed as:





1. **Kalman filter**

Prediction process:



Update parameters:



**Wheeled-Legged Odometry**

In the structure of the multi-legged robot, a wheel rotation degree of freedom is added to the end of the multi-legged robot branch, and the contact with the ground is a non-integrity constraint.

1. **State prediction model**

In order to better describe the fuselage position state of the quadruped wheel-legged robot, the absolute position of the body is decomposed into the absolute position accumulated by the leg motion and the absolute position accumulated by the wheel motion, so the state variable is selected as:



where  represents the absolute position and linear velocity of the body,  represents the absolute position of the foot， mean the part generated by the wheeled motion and the legged motion.

According to the definition, the absolute position and linear velocity accumulated by the row motion of the wheel leg can be obtained:



The fuselage acceleration measured by the IMU sensor is the input of the prediction model:



where  is the linear acceleration of the IMU， is the gravity。

Prediction model can be expressed as：



where ，



 are the noise coefficient.

1. **Observation model**

Observation variable：



where  are the position and linear velocity obtained by the extern sensor.

1.  can be obtained by the forward kinematics of one leg.
2.  represent the body velocity observed by the leg and wheel motion of the supporting leg. The observation is invalid when the leg is swing. The body velocity obtained by the legged motion can be computed by the forward kinematics of one leg, and the body velocity obtained by the wheeled motion can be computed by the wheel velocity and the forward kinematics of one leg.

Observation equation is:



where 

Noise:where are the noise coefficient.

1. **Model optimization**

Whether a single leg supports the ground face has a great influence on the state estimation of a multi-legged robot during walking. A support confidence parameter  was designed according to the gait design and leg branch motion. Under ideal conditions, the confidence parameter of support state is 1, and the confidence parameter of swing state is 0.

Covariance and observed variables are adjusted for confidence parameters:

* 1. Optimization of observation variables

The observed body velocity computed by the legged motion is not accurate, and the observed velocity can be modified as:



where  is the part of the body velocity obtained by the wheeled and legged motion.

* 1. Optimization of covariance

When the end of the leg branch is in the supporting state, the reliability of all state prediction and observation is high, but the confidence of the end position predicted by the swing state is low, and the confidence of the observed end velocity and height is very low.

Assuming a constant noise , a class of common matrices is constructed:



Then the two noise optimizations can be expressed as:





1. **Kalman filter**

Prediction process:



Update parameters:

